

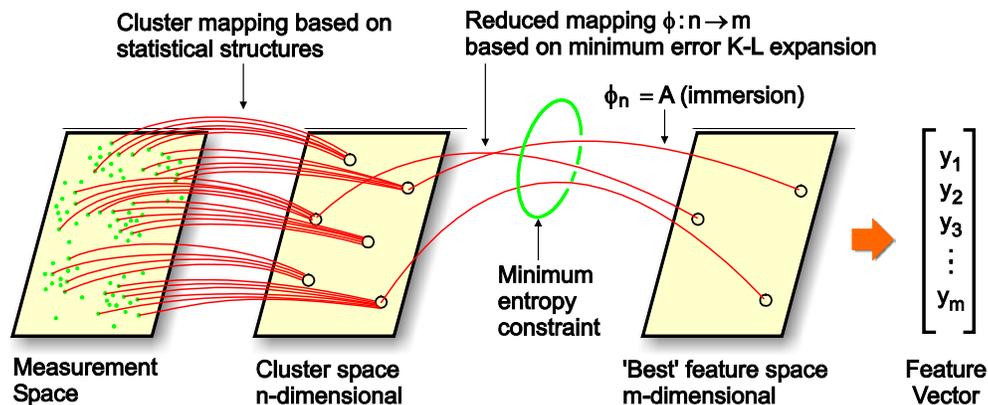
The Nature of Bell's Hidden Constraints

Abstract

Viewing *Math* and *Physics* as Korzybski's 'map' and 'territory', we question their trust and tricks. Maps derived from observations of the real world bring *eigenvalue-based measurement* into question. But what to do when the map logic conflicts with our physical intuition? This is often resolved in favor of the non-intuitive, whether *simultaneity* in relativity or *non-locality* in the case of Bell's theorem. The subtle nature of Bell's hidden constraints *erasing* the hidden variable information is the basis of Bell's lack of trust in his intuition.

The physical world does exist, as anyone can prove by jumping from a high place. Almost 400 years ago Newton predicted the behavior that follows such a jump, based on math developed in his mind. We use our minds to connect math and physics and our intuition to judge the results. Math arises in the physical world, not the other way around. In Korzybski's sense, math is the *map* and the physical world is the *territory*.

Although the essence of math is awareness of relations and patterns, it is possible to structure physical machines in such a way as to perform mathematical operations, with the fundamental operation being *counting*. Once a counter produces a number, another machine can add (subtract) this number to a different number to yield a new number. If the new number is zero, an identity of some kind is established, because identity implies zero 'distance' from an entity. Distance, as difference, is a key concept leading to pattern recognition machines that measure distances between points in a set of measurements and perform inter-set and intra-set operations to partition and group sets into subsets, as diagrammed below. This feature extraction from measurements allows us to partition a real physical continuum into a feature-based world. Math operations on real world *features* are the basis of the science of physics.



In *The Automatic Theory of Physics* [1], I describe the concepts and techniques represented in the above diagram that shows the transformation of measurement data into *best* feature vectors, and the dynamical processes that produce eigenvalues,

generally taken to be truly representative of the object system.

FQXi asks why math is so 'unreasonably' effective in fundamental physics.

The above feature extraction is based on *distances* obtained from these simplest math operations, and these math operations are easily constructed from physical structures (atoms, molecules, DNA, proteins, cells, organisms, neural nets, and logic machines) that can function as gates, implementing AND and NOT logic operations, which can be combined to *count* to produce integers and to *add* to produce *distance* maps and then compare distance maps to get *difference* maps (gradients) from measurements. The nature of the process of making math maps is thus rooted in the physical universe.

Math maps imposed on the physical territory form the substance of physics.

FQXi asks whether these maps can trick us, or whether we should trust the maps.

Specifically, what should we do when map logic conflicts with our physical intuition? I believe that the physical world can be trusted. I have awareness of only *one* physical universe, but I have *many* maps of the universe, and I trust that I can use the physical universe to qualify the maps. The key feature of the universe that allows this approach to work is the repeated observability of the physical world, which is closely related to the stability of object systems. We typically construct a formal analog of the physical object systems of interest using mathematical objects known as state vectors. These are based on partitioning the universe of repeatedly observable phenomena into object systems with properties (including dynamic properties) derived from measurements.

While (math) maps of the (physical) territory form the substance of physics, one must apply the *right* map at the *right* place. A New Orleans map applied to Chicago only goes so far before one loses trust. To begin, we discard maps that do not represent territory. Multiverse maps point to no observed territory. Nor do string maps. Maps based on eigenvalue landmarks are trickier. Such a map, based on repeatable observations, combined with eigenvalue representations, can trick physicists into not trusting their intuition. The eigenvalue problem is complex [2], and, as HL Mencken said:

"Complex problems have simple, easy to understand, wrong answers."

Bell demonstrated this by following his maxim [3]:

"Always test your general reasoning against simple models",

while ignoring Einstein's maxim: *"... as simple as possible, but not simpler."*

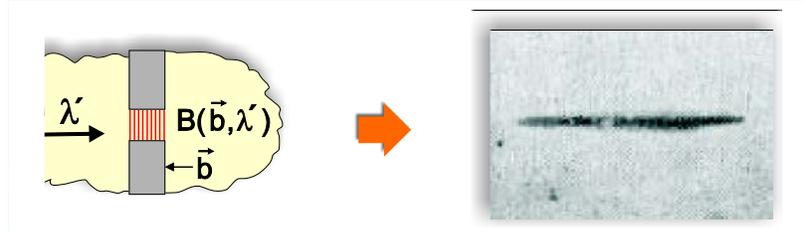
FQXi then asks, *"Why do we prefer mathematically simple theories to complex ones?"*

The answer may be aesthetic, or it may be Occam, but Mencken suggests that the answer may not be simple. This suggests the possibility that when a map conflicts with intuition, it may be the wrong map. I explore this idea in terms of *eigenvalue maps* and distinguish between two spin eigenvalue maps [4] that I believe most physicists have assumed to be essentially identical. I show that the maps differ.

FQXi also asks, *"Are there hidden subtleties in how math is used in physics?"*

I believe that one such subtlety is that Bell's focus on 'hidden variables' has managed to keep his 'hidden constraints' off the radar for 50 years! [20]

“Theoretical physics is all about writing down models to describe the behaviors of particular systems in the Universe [21].” Bell [6] based his model on EPR [7], i.e., on the Stern-Gerlach experiment, which sends neutral particles with spin-based magnetic moments through magnetic fields. Two cases are special: particles in a *constant field* precess — a spin λ moving in a constant field simply precesses and is not deflected.



But the same particles moving through an inhomogeneous field are deflected by the gradient force $\vec{F} = \nabla(\vec{\mu} \cdot \vec{B})$ where $\vec{\mu}$ is the magnetic moment ($\sim \lambda$) and is represented by quantum operator $\hat{\sigma}$, and $\vec{B}(\vec{x})$ is the magnetic field.



The data shown are from the iconic postcard Stern-Gerlach sent to Bohr announcing their discovery of the spin-dependent 'splitting'. Prior to Bell, per Messiah [8]:

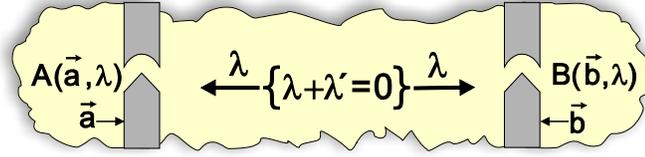
“The appearance on the screen of a more or less spread out distribution of impacts indicates that the atoms are not all in the same initial condition and that the dynamical variables defining the initial states are statistically distributed over a somewhat extended domain.”

Bell analyzed the case in which two particles are prepared with opposite spins and sent through differently oriented B-fields, in which orientation \vec{a} is freely chosen by 'Alice' and orientation \vec{b} is freely chosen by 'Bob', and neither one knows the other's choice. If Alice's measurement of spin λ is denoted by $A(\vec{a}, \lambda)$ and Bob's by $B(\vec{b}, \lambda)$, the expectation value (probable average) over many experiments is specified by QM

$$\langle AB \rangle = \langle \text{singlet} | \hat{\sigma}_A \cdot \vec{a} \hat{\sigma}_B \cdot \vec{b} | \text{singlet} \rangle = -\vec{a} \cdot \vec{b}. \tag{1}$$

But quantum mechanics is a statistical theory and cannot predict the outcome of either Alice's or Bob's experiment, only a statistical product over many such. Bell asked if there is a more detailed underlying classical model that can predict individual outcomes as well as the statistical correlations [6]. He concluded that it is *impossible* for any local realism (or local causality, or local deterministic) model to produce the QM correlation, $-\vec{a} \cdot \vec{b}$. The situation is schematically shown as described by Gisin[9]:

"... quantum physics tells us it is possible and even commonplace for two widely separated objects in space to form in reality a single entity...! And that's entanglement."



Bell's key physical assumptions [6] that led to this conclusion were:

1. Stern-Gerlach measures spin components.
2. The spin eigenvalue equation is $\sigma_z |\pm\rangle = \pm |\pm\rangle$.
3. The relevant physical force, $F \cos \theta \Rightarrow F \cos \theta / |\cos \theta|$.
4. The magnetic moment precesses as it traverses the magnetic field.

When Bell replaced the classical force on a magnetic moment, $F \cos \theta$, with the force $F \cos \theta / |\cos \theta|$ he suppressed the physics of the classical model described by

$$E = -\vec{\mu} \cdot \vec{B} + \vec{\nabla}(\vec{\mu} \cdot \vec{B}) \cdot d\vec{x} \quad (2)$$

Bell claimed the suppression of θ was just an *ad hoc* attempt to account for observations. But if total energy is conserved and two components depend on a common physical variable, a system is unstable, and undergoes change as a function of the unstable parameter, θ . Eqn (2) describes a system with an energy mode that depends upon the local magnetic field, and another energy mode that depends on the gradient of the magnetic field; the term in common connecting the two physical modes is the angle $\theta = \cos^{-1}(\vec{\mu} \cdot \vec{B} / |\vec{\mu}| |\vec{B}|)$. Thus total energy is $E = -f(\theta) + \vec{g}(\theta) \cdot d\vec{x}$. If the system energy is conserved, $dH/dt = 0$, we state [2] and prove the

Energy Exchange theorem:

If a physical system possesses two energy modes, M_0, M_1 , coupled to a common variable θ , and energies of the modes are not separated by a quantum gap $\Delta \varepsilon > 0$, then if the common variable changes, $d\theta/dt \neq 0$, the modes will exchange energy.

Assume that total energy is $\varepsilon = \varepsilon_0 + \varepsilon_1$ when $H_i |\psi\rangle = \varepsilon_i |\psi\rangle$ and that total energy $H = H_0 + H_1$ is conserved:

$$\frac{dH}{dt} = 0 \Rightarrow \frac{dH_0}{dt} + \frac{dH_1}{dt} = 0 \Rightarrow \frac{dH_0}{d\theta} \frac{d\theta}{dt} + \frac{dH_1}{d\theta} \frac{d\theta}{dt} = 0 \Rightarrow \left(\frac{dH_0}{d\theta} + \frac{dH_1}{d\theta} \right) \frac{d\theta}{dt} = 0$$

Since $d\theta/dt \neq 0$ then
$$\frac{dH_0}{d\theta} = -\frac{dH_1}{d\theta} \quad (3)$$

and energy flows between mode M_0 and M_1 . QED

In the Stern-Gerlach experiment these energy modes depend on the variable θ and hence exchange energy by the energy-exchange theorem. By modifying force $F \cos \theta$ Bell intentionally removed the θ -dependence, and he essentially removed the energy exchange physics from the problem. We restore the physics of θ to a classical model by assuming a random particle spin before it enters the magnetic field and we predict the position of the particle after leaving the magnetic field. Both data and analysis suggest that deflections are not quantized; the particle is governed by energy equation, $E = -\vec{\mu} \cdot \vec{B} + \vec{\nabla}(\vec{\mu} \cdot \vec{B}) \cdot d\vec{x}$. Our energy-exchange model [2] will exchange θ -dependent precession energy with deflection mode energy, leading to a variable deflection with an x-direction contribution:

$$x = \left(\frac{|\vec{\mu}| |\vec{B}|}{|\nabla(\vec{\mu} \cdot \vec{B})|} \right) (1 - \cos \theta). \quad (4)$$

Earlier, we stated that: "*the eigenvalues are generally taken to be truly representative of the system.*" In *The Formalisms of Quantum Mechanics* [10] the expectation value of an observable (the outcome of a measurement on the system in state ψ),

$$\langle A \rangle_{\psi} = \langle \psi | A | \psi \rangle = \langle \psi | A \psi \rangle, \quad (5)$$

"implies ... that the possible outcomes of the measurement of A must belong to the spectrum of A, i.e., can only be equal to eigenvalues of A."

This is what Bell believes he is assuring by constraining measurement values:

$$A(\vec{a}, \lambda) = \pm 1 \text{ and } B(\vec{b}, \lambda) = \pm 1. \quad (6)$$

Bell concluded that the ultimate nature of reality is non-local, contrary to intuition. Based on his assumptions, Bell showed *a local model cannot produce* the correlation $-\vec{a} \cdot \vec{b}$, while our analysis of Stern-Gerlach experiments has led to construction of *a local model that does produce* correlation $-\vec{a} \cdot \vec{b}$, based on energy-exchange physics that Bell suppressed. Our model has been challenged [11] by some as follows:

- A. Electron spin has two eigenvalues
- B. Idealized experiments yield eigenvalue measurements.

Further, Dirac's equation derives dichotomous spin, so A implies B.

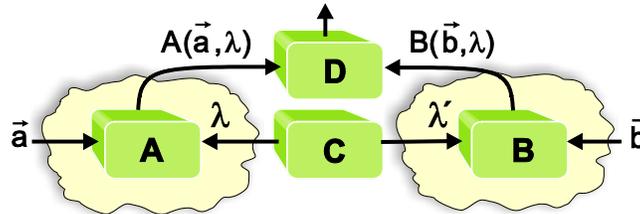
This challenge may sound logical, but an extensive analysis by Potel [12] states, in 'Quantum mechanical description of Stern-Gerlach experiments':

"Thus, we can conclude that the Stern-Gerlach experiment is not, even in principle, an ideal experiment, which would "project" the internal state into the eigenvalues of the measurement operator."

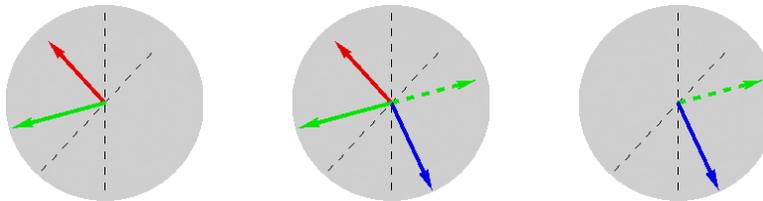
Belief in *eigenvalue measurement*, as sufficient reason for constraints $A, B = \pm 1$, is challenged quantum-mechanically by Potel's paper. We challenge it further after developing a local model that *does* succeed in violating Bell's theorem.

Our local realism model

In the local model the initial spin λ (hidden from QM) makes angle $\theta = (\vec{a}, \lambda)$ when Alice chooses \vec{a} as the direction of her Stern-Gerlach magnetic field; she will calculate a scattering angle with a component given by eqn (4). Bob will see initial spin $\lambda' = -\lambda$ with angle $\theta' = (\vec{b}, \lambda')$ and calculate the local deflection predicted for his SG apparatus. Two values of deflection representing measured outputs can, post-experiment, be processed by a decision module D to generate the correlations. The system based on our local classical model is shown below



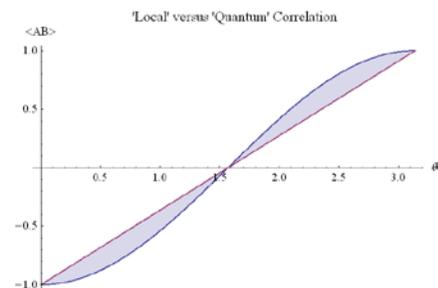
The model generates random local spins $\lambda + \lambda' = 0$ then sends λ to Alice's A-module and λ' to Bob's B-module. Alice chooses a random direction \vec{a} and calculates the θ -dependent contribution to deflection from eqn (4). An example of randomly generated vectors for each experiment is shown below, with spin λ (green), λ' (dashed green), and vectors \vec{a} (red) and \vec{b} (blue).



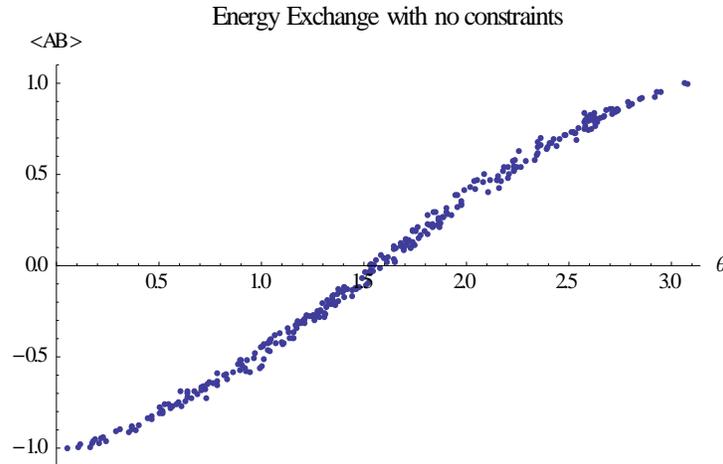
Recall that actual experiments show that the correlated measurements agree with the quantum mechanical prediction, and the consensus of physicists is that:

"No local model can reproduce QM prediction"

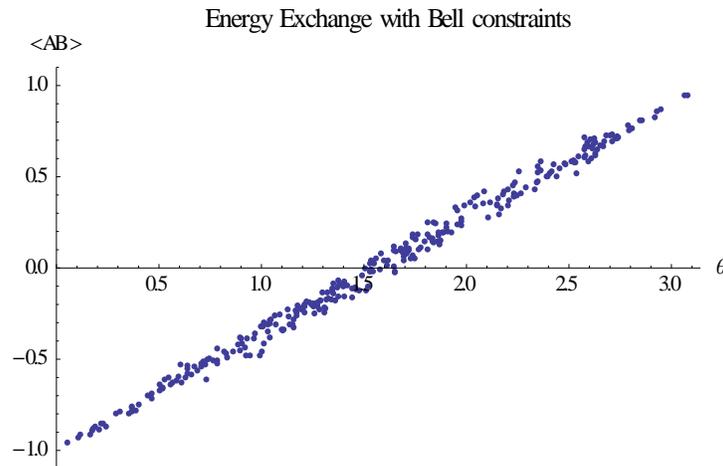
Peres [13] describes quantum correlation, $-\vec{a} \cdot \vec{b}$ and classical correlation, $-1 + 2\theta/\pi$, seen at right, where we have shaded the difference between these curves. In the next figure below we show that the correlation derived from a local model based on *energy-exchange physics* clearly violates Bell's straight line prediction.



As described above, the local model generates random values of spin and of the control settings chosen by Alice and Bob and sends them to their respective modules, where θ -dependent deflections are calculated. The next figure shows the correlated results of our local classical model generating 10,000 random particle spins λ for *each* of 300 random pairs of control settings, \vec{a} and \vec{b} :



This local realism result exactly agrees with the quantum prediction that Bell claimed to be impossible for local models. How can this be? Let us compare this to results obtained from the same local model with Bell's constraints applied:



Let us clearly understand the difference in the two models. My local model with *energy exchange physics* reproduces the correct correlations which Bell claims to be impossible, achieved by calculating the θ -dependent outputs from Alice and Bob. Bell, having suppressed the θ -physics, cannot calculate these values of deflection. If I throw away this θ -information by truncating the measurement data, i.e., setting the results to $A, B = \pm 1$, my constrained model *cannot* produce the correct correlations. No local model of Bell's can reproduce QM correlations because he applies the hidden constraints that *erase* the hidden variable information. Yet Bell's many followers are adamant that one must apply Bell's constraints. They believe strongly that 'spin' is being measured, that spin has eigenvalues ± 1 , and that *ideal measurements yield eigenvalues*. It is relevant that Peres [13] states that

“Bell’s [paper] is not about quantum mechanics. Rather, it is a general proof, independent of any specific physical theory, that there is an upper bound to the correlation of distant events, if one just assumes the validity of the principle of local causality.”

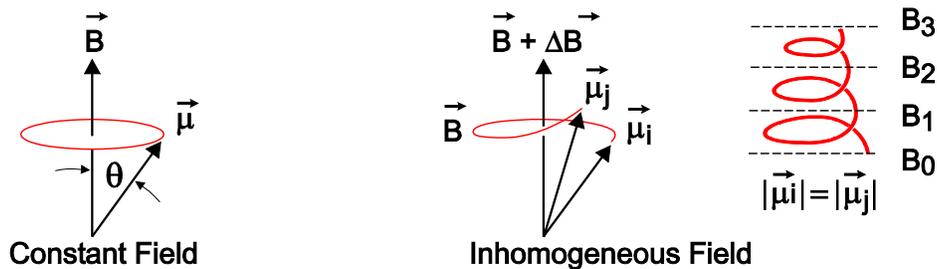
Peres is mistaken. Bell's paper is about quantum mechanics. There is absolutely no other reason to impose the constraint $A(\vec{a}, \lambda) = \pm 1$ on the results of measurement. It is a quantum mechanical argument Bell's defenders make concerning eigenvalues.

The Map Trick Revealed

This is where the 50 year old map trick enters the picture. To see it, let us look at the energy-exchange physics. The following diagram shows precession in a constant field on the left. While this is described by an eigenvalue equation (the 'map' Bell chooses),

$$\hat{\sigma} \cdot \hat{B} |\pm\rangle = \pm |\pm\rangle, \tag{7}$$

it produces no Stern-Gerlach results, thus leading to a contradiction. The precessing spin on the right is moving in an inhomogeneous field, so the angle θ is changing. After a number of cycles, the spin aligns with the B-field and precession terminates. So the precessing eigenvalue equation is provisional, *provided* there is a constant field; Schrödinger's equation [14] then has a $\hat{\sigma} \cdot \vec{B}$ term and energy eigenvalues $\pm \hbar\omega/2$.



We conceive of the physical phenomena in terms of the two energy states of a *quasi-particle*, consisting of the interactions of the particle with the local field. But when the local field is inhomogeneous, the θ -dependent force yields energy term $\sim \vec{\nabla}(\vec{\mu} \cdot \vec{B}) \cdot d\vec{x}$, and Schrödinger's equation produces a *continuous* eigenvalue spectrum, representing the θ -dependent deflections. This fundamentally differs from Dirac's relativistic four-component eigenvalue equation. An inhomogeneous field can be regarded as a perturbation, which can cause transitions (at least virtual ones) between states with energies of opposite signs, so that a mixing of components seems to be inherent in the Dirac wavefunction. Messiah in *Quantum Mechanics* [15] notes:

Due to the coupling between the positive and negative components of the four-component Dirac wave function, [Messiah's equation (XX.183)] is, properly speaking, no longer an eigenvalue equation.

This motivates the Foldy-Wouthuysen transformation, which allows one to approximate the four-component Dirac theory by a two-component theory to any order in v/c , and thus remove the redundancy. In the Dirac representation, the orbital angular momentum $\vec{r} \times \vec{p}$ and the spin angular momentum $\sigma/2$ are not separately constants of the motion, although their sum is. After the Foldy-Wouthuysen transformation [16] these are decoupled and are separately constants of the motion. At this point the transformed operators representing physical quantities are in a one-to-one correspondence with the operators of the Pauli theory, thus linking the Dirac relativistic

theory to the Pauli nonrelativistic theory used by Bell. The transformation produces a fundamental *helicity* equation (map), $\hat{\Sigma} \cdot \hat{p} |\pm \vec{p}\rangle = \pm |\pm \vec{p}\rangle$, which, formally, *looks like* the provisional precession eigenvalue equation (map), but is quite different [17].

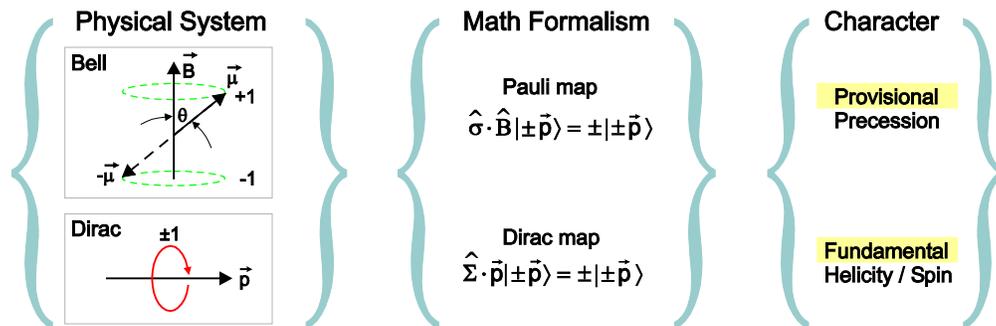
Dirac eigenvalue map	or	Pauli eigenvalue map
$\hat{\sigma} \cdot \hat{p} \pm \vec{p}\rangle = \pm \pm \vec{p}\rangle$	\oplus	$\hat{\sigma} \cdot \hat{B} \pm \vec{p}\rangle = \pm \pm \vec{p}\rangle$

where $\hat{p} = \vec{p} / |\vec{p}|$ and $|\pm \vec{p}\rangle = |\pm\rangle \otimes |\vec{p}\rangle$ with $|\pm\rangle =$ intrinsic angular momentum eigenvector and $|\vec{p}\rangle =$ linear momentum eigenvector. By imposing the constraints $A(\vec{a}, \lambda) = \pm 1$ and $B(\vec{b}, \lambda) = \pm 1$, Bell believes he is measuring fundamental spin, in the sense of Dirac, rather than the provisional quasi-particle precession that applies to Stern-Gerlach. Bell is simply using the *wrong eigenvalue map*. He tricked himself by

"testing his general reasoning against too simple a model."

Conclusion

Our analysis shows Bell's theorem to be oversimplified. Bell's assumptions led to his claim that it is impossible for any local deterministic model to reproduce the quantum prediction, $-\vec{a} \cdot \vec{b}$. But the eigenvalue analysis of Bell's hidden constraints (*hidden*, because not recognized as constraints by physicists) implies that Bell confused Pauli's *provisional precession eigenvalue equation* with Dirac's *fundamental helicity eigenvalue equation*. The FW-Dirac equation is assumed *always* true, but provisional equations are true *only provided* that the local magnetic field is constant, and this leads to a contradiction, thus bringing Bell's claim of non-locality into question.

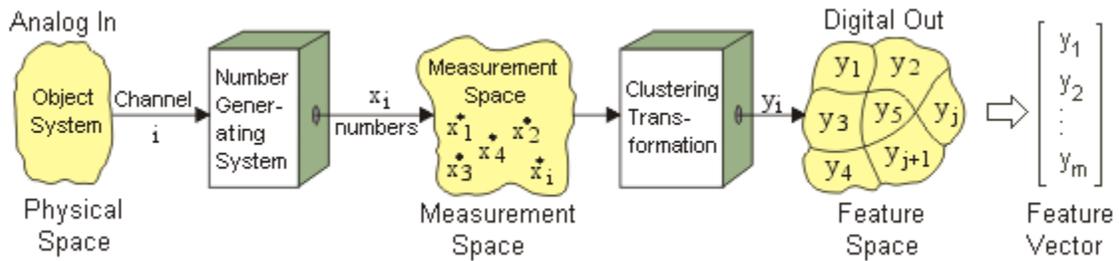


The results of an unconstrained energy-exchange model of local physics agree perfectly with the experimentally verified quantum predictions; while the same model with Bell's constraints applied fails to do so. Bell's 50 year old *proof* of the non-local nature of the Universe is an over-simplified solution to a complex problem. As this is generally considered the basis of 'entanglement', it suggests that reappraisal of much of current physics is in order. However, θ -dependent scattering should be testable experimentally. If experimental tests of θ -dependence agree with our energy-exchange theory, the 50 year old belief in non-intuitive non-locality will be seen as a consequence of overly simple assumptions leading to the improper imposition of hidden constraints.

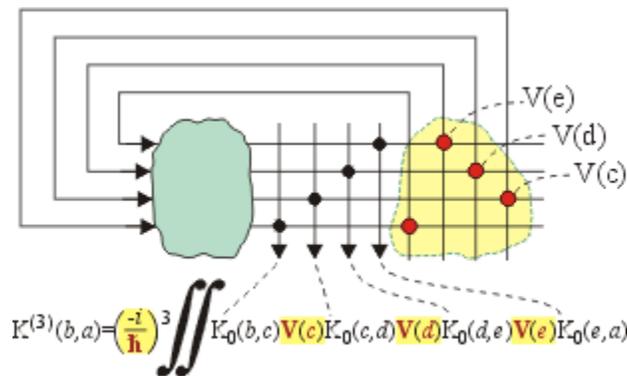
Bell simply applied the wrong map to the territory.

Endnotes

The following is an alternative view of the transformations on page 1 of the essay:



In *The Automatic Theory of Physics* [1] I state: “All axiomatized theories of physics can be formally mapped into automatic machine representation.” A typical 3rd order Feynman diagram¹ quantum field theory kernel $K^{(3)}(b, a)$, shown below in canonical form, maps the automata’s ‘Next State-address’ into ‘local potential’ in physics.



The issue of eigenvalues and eigenstates

Searching for a local explanation of quantum correlations, Bell chooses *eigenvalues* as the meaning of $A(\vec{a}, \lambda) = \pm 1$. Eigenvalues are concepts defined by quantum operators operating on quantum states. But what is a quantum state? In 2014 we simply don't know. Leifer states [18]:

The status of the quantum state is one of the most controversial issues in the foundations of quantum theory. Is it a state of knowledge (an epistemic state), or a state of physical reality (an ontic state)?

An ontological model for [...] experiments is an attempt to explain the quantum prediction in terms of some real physical properties... that exist independently of the experimenters,

while epistemic or 'knowledge' models lead to such questions as "what is precessing?" and to concepts such as *collapse of the wave function*. Einstein's belief was in "real physical properties" that exist independently, and our local model assumes the same.

Review of Quantum Mechanical model of spin

We review Susskind's [14] nonrelativistic treatment of spin: Define a 2-dimensional spin state vector, $|u\rangle$ and $|d\rangle$ (*up* and *down*) with representation:

$$|u\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad |d\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

"We know σ_z has definite, unambiguous values for the states $|u\rangle$ and $|d\rangle$, and that the corresponding measurement values are $\sigma_z = +1$ and $\sigma_z = -1$."

Here σ_z is the operator corresponding to the spin observable. Susskind states:

"The eigenvectors of σ_z are $|u\rangle$ and $|d\rangle$. The corresponding eigenvalues are +1 and -1. We express this with the abstract equations

$$\sigma_z |u\rangle = +|u\rangle \quad \sigma_z |d\rangle = -|d\rangle \quad (3.12)$$

We combine these into

$$\sigma_z |\pm\rangle = \pm |\pm\rangle$$

This eigenvalue equation recalls the time-independent Schrödinger equation,

$$H |E_j\rangle = E_j |E_j\rangle$$

where H is the Hamiltonian operator, and the observable values of energy are just the eigenvalues, E_j , of H , with corresponding eigenvectors $|E_j\rangle$. For a magnetic field along the z -axis, the Hamiltonian is proportional to σ_z :

$$H = \frac{\hbar\omega}{2} \sigma_z \sim \vec{\sigma} \cdot \vec{B} = \sigma_x B_x + \sigma_y B_y + \sigma_z B_z$$

But, as we have seen, the eigenvalues that flow from this Pauli model are not the same as the helicity eigenvalues that Dirac discovered.

$$\hat{\sigma} \cdot \hat{p} |\pm \vec{p}\rangle = \pm |\pm \vec{p}\rangle \quad \hat{\sigma} \cdot \hat{B} |\pm \vec{p}\rangle = \pm |\pm \vec{p}\rangle$$

Bell claims that the measurement results for any hidden variable model must be ± 1 . He does so based on a model in which he has suppressed the relevant physics and assumed that the spin precesses as it traverses the apparatus. He expresses this requirement as $A(\vec{a}, \lambda) = \pm 1$ and $B(\vec{b}, \lambda) = \pm 1$. As a result [19],

"Bell proved that some predictions of quantum mechanics cannot be reproduced by any theory of local variables."

Our emphasis is on *any*. The literature is full of such general statements, with absolutely no qualifications. But one could qualify... Bell's defenders do not really mean "any" theory — they mean only any theory constrained to produce ± 1 results.

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Note: All URL's verified on 27 Dec 2014

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