

Article

A Primordial Space-time Metric

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Abstract

Pre-space-time reality is modeled as a primordial field, a matterless continuum or superfluid, in which local field energy is effectively limitless. Super-fluids are characterized by vorticity $\kappa = \nabla \times b$ where \vec{b} is magnetic-like: $\nabla \cdot b = 0$, $b = \nabla \times A$. The problem of pre-space-time modeling is the *evolution* of the system: Unlike the Schwarzschild static solution $g_{00} = -(1 + 2\phi)$, the pre-space-time metric must evolve with time. The *Horizon Problem* seeks to explain the isotropy of the universe based on inflation models wherein separate regions of the universe are never in contact; therefore a solution should hold at arbitrary points in the field. In this paper, I construct the physics of h_{ij} for a dynamic spatially homogeneous anisotropic Bianchi vacuum model that exactly solves Einstein's equations in terms of the physically real primordial field.

Keywords: Primordial field, spacetime metric, prespacetime, superfluid, horizon problem, universe, gravitation.

[U]nlike all other fields in nature which live in space-time, in essence the gravitational field is space-time."Cooperstock GRF-2017

Introduction

The *Cosmological Principle* is a generalization of the *Copernican Principle* that Earth is *not* the center of the solar system; leading to the requirement of both isotropic and homogeneous universe. In reality there are few grounds to support the cosmological principle¹⁰; it is rather a cultural response to the historically significant rejection of *Earth as center*, designed to preempt physicists from making the same mistake on a larger scale. Homogeneity and isotropy are statistically 'smeared' as they obviously fail on smaller scales. At a larger scale the *cosmic microwave background* provides support for isotropy, however there are experimental and theoretical grounds for investigating different models. D'Inverno notes⁶ that "*calculations of statistical fluctuations in Friedmann models suggest that they cannot collapse fast enough to form the observed galaxies.*" The 2004 WMAP¹³ *axis-of-evil* and its 2013 *Planck Telescope* confirmation¹⁴, combined with Longo's¹² surveys of *spiral galaxy orientations* in both Northern and Southern hemispheres, suggest rotation unaccounted for by the cosmological principle.

Theoretical investigations of anisotropic and inhomogeneous solutions of Einstein's equations include Bianchi models: spatially homogeneous anisotropic models. General relativistic curved

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space-time does *not* support localized conserved quantities like energy: "*it is impossible to construct unique covariant densities of these quantities.*" On the other hand, Petrov¹¹ observes that:

[Space-time] *is an arena, in which physical fields interact and propagate;*
 (...) *The space-time itself is a dynamic object.*

If we wish to model the primordial field as a dynamic field that expands and contracts in different directions, we seek a solution of Einstein's equations, hence a metric solution. Einstein¹ believed that "*Space-time does not claim existence on its own, but only as a structural quality of the field.*" Yau² proposed: *gravitational fields exist without matter*, and Feynman¹⁰ noted that, if time dependence is allowed, "*the equations permit the presence of fields without sources*".

General relativity is defined by $ds^2 = g_{\mu\nu}dx^\mu dx^\nu$, where curved *empty* space coordinates dx^μ and dx^ν correspond to flat-space coordinates $\eta_{\mu\nu}$. Einstein's equation $G^{\mu\nu} = 8\pi T^{\mu\nu}$ is interpreted to mean that a stress-energy tensor $T^{\mu\nu}$ induces 'curvature' in the geometry of space-time, where $T^{\mu\nu}$ represents energy density or matter. The simplest case, for spherically symmetric mass M , has curvature given by the Schwarzschild metric:

$$ds^2 = -e^{2\phi} dt^2 + e^{-2\phi} dr^2 + r^2 d\Omega^2, \tag{1}$$

which specializes $ds^2 = g_{\mu\nu}dx^\mu dx^\nu$ to $g_{00} \approx -(1+2\phi)$, $g_{11} \approx (1+2\phi)^{-1}$, where ϕ reduces to Newtonian gravity $\phi = -M/r$ in the appropriate limit. This metric leads to a singularity at $g_{11} \approx (1+2\phi)^{-1}$ with the well-known black hole horizon associated with radius $r = 2M$; the Kerr metric describes the field for spinning M . Nevertheless, important cosmological solutions are also found when $T^{\mu\nu} = 0$, i.e. there is no stress energy term representing either matter or distributed energy, thus leading to $R^{\mu\nu} = 0$. For example the *deSitter* metric solves Einstein's equation with cosmological constant, but is otherwise *devoid of matter*. The solution looks the same in any direction, hence isotropic; however our interest is in *anisotropic* models; such a solution to $R^{\mu\nu} = 0$ was discovered in 1921 by Kasner, and rediscovered in alternate form in 1946 by Narlikar and Karmarkar and by Taub in 1951. The linearized metric

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \tag{2}$$

describes most of the *non-black-hole* universe adequately. The gravitational field is described in general relativity via space-time metric: $h_{00} = -2\phi$

$$ds^2 = -(1+2\phi)dt^2 + (\delta_{jk} + h_{jk})dx^j dx^k + 2h_{0j} dt dx^j \tag{3}$$

where $\phi(x, y, z)$ is the time-independent Newtonian potential M/r in geometricized units $g = c = 1$. The *Schwarzschild* static solution is described (outside of M) by variants of (3), and by *Kerr* if the mass is spinning. Massless solutions exist: *deSitter*, *TaubNUT*, and *Ozsvath-*

Schucking,³ including Kasner's exact solution for $D > 3$ space-time dimensions, in Narlikar and Karmarkar formulation:⁴

$$ds^2 = c^2 dt^2 - \sum_{j=1}^{D-1} (1 + nt)^{2p_j} dx_j^2 \quad (4)$$

subject to $\sum_{j=1}^{D-1} p_j = 1$ and $\sum_{j=1}^{D-1} p_j^2 = 1$.

Per Vishwakarma⁷ the conventional Kasner metric interpretation is “*obscure and questionable*”.

The Physical Interpretation of the Kasner Metric

Kasner appears to be an appropriate metric for the primordial field, so we investigate the physical interpretation of the metric. Vishwakarma observes that when $T^{\mu\nu} = 0$ there is no obvious source of curvature in the vacuum; the mass that sources the Schwarzschild and Kerr solutions does *not* exist in the matterless primordial field. He also notes that when $n = 0$, metric (4) reduces to the Minkowski metric, therefore the source of curvature must be contained in n . But what is n ? We assume that nt is dimensionless, so that it can be added to the number 1, hence n has dimensions $1/t$. Vishwakarma discusses the 'singularity' for this metric when $n = 1/-t$ (denoting negative time or frequency.) But this one-time occurrence of the singularity, unlike the '*always there*' Schwarzschild radius in space, vanishes after the singular moment; nothing is left to sustain space-time curvature contained in the metric.

Recall that the metric represents 'dynamic space-time', while Einstein^{1,155} believed that "*there is no such thing as empty space, i.e. a space without field.*" Therefore dynamic space-time corresponds to a dynamic field, and the energy density of the field implies an equivalent mass density 'in motion'. Vishwakarma thus identifies *the momentum density of the field* as the source of metric curvature, and associates the momentum density with $\vec{p} \sim (p_1, p_2, p_3)$. At this point we have the Kasner metric associated with two physical properties: frequency n and momentum density $\vec{p} = \frac{\vec{P}}{\int d^3r}$. This is progress, but the interpretation is still obscure. It is possible in

physics to derive mass, length, and time from universal constants (g, c, \hbar) as follows (Planck length $\Delta x = \sqrt{\hbar g / c^3}$, mass $\Delta m = \sqrt{\hbar c / g}$, time $\Delta t = \sqrt{\hbar g / c^5}$). Vishwakarma chooses to link the momentum density $\vec{p} = \rho \vec{v}$ to frequency $n = t^{-1}$ via the universal constants as follows: [Newton's gravitational constant g , speed of light c]

$$n = \sqrt{\frac{g\bar{p}}{c}} = \sqrt{g\rho\left(\frac{v}{c}\right)}$$

Note that (v/c) is dimensionless and $g\rho = \left(\frac{l^3}{mt^2}\right)\left(\frac{m}{l^3}\right) = \frac{1}{t^2}$ therefore

$$n = \sqrt{g\rho\left(\frac{v}{c}\right)} = t^{-1} \quad (5)$$

Interesting as this relation is, it is still obscure. It relates *momentum density* to *frequency* as the physical correlates of the metric, but the physical meaning of frequency n is still unknown. We choose to introduce a relevant characteristic of the primordial field as follows⁹:

$$\vec{C} = \left(\frac{g}{c^2}\right)\vec{p} \times \frac{\vec{r}}{r^3} \sim t^{-1} \quad (6)$$

This offers the advantage of transforming the basis of the Kasner metric from a nebulous frequency, formed of a well-chosen combination of universal constants, to the well-known gravito-magnetic field \vec{C} induced by momentum density \vec{p} at distance \vec{r} from \vec{p} according to the circulation equation

$$\vec{\nabla} \times \vec{C} = -\left(\frac{g}{c^2}\right)\vec{p} \sim (lt)^{-1}. \quad (7)$$

It seems to have gone unrecognized to date that the parameter n thus linked to momentum density \vec{p} has the dimension of the gravitational C-field and the dynamical relation between the two

$$\vec{\nabla} \times \vec{C} = -(g/c^2)\vec{p} + c^{-2}\partial_i \vec{G}, \quad (8)$$

can be obtained from linearizing Einstein's nonlinear field equations. The misnomer *weak field equations* falsely characterizes the field strength dependence. The C-field circulation law applies at *any* field strength and is iteratively equivalent to the full nonlinear equations. The common belief that the linearized field equations are *weak field* equations might lead one to wonder whether they apply in the *ultra-strong fields* of the early inflationary era; however, the complete equivalence of the linearized equations, iteratively applied, to the full non-linear field equations can be shown. The presence of static mass M leads to a gravitational field: $\vec{G} \sim -\vec{\nabla}\phi$, whereas the presence of momentum density \vec{p} leads to the gravitomagnetic field circulation: $\vec{\nabla} \times \vec{C} \sim \vec{p}$. Gravitational fields have units of acceleration $m\vec{a} = m\vec{G}$ with $|\vec{G}| \sim l/t^2$ while the gravitomagnetic field has units of angular frequency $\sim 1/t$. Thus Kasner parameter n , with dimension $\sim 1/t$, will be identified with the gravitomagnetic field \vec{C} where $|\vec{C}| = 1/t$. The primordial C-field comports with Kerson Huang's¹⁴ *Superfluid Universe*, wherein vorticity $\vec{\kappa} = \vec{\nabla} \times \vec{b}$ is the characteristic feature. The constraints $Tr(\vec{p}) = 1 = \vec{p} \cdot \vec{p}$ on p_i describe a *plane* and a *sphere* in an anisotropic universe without matter; a *vacuum* solution. Static solutions are isotropic and homogeneous, while Kasner's solution is homogeneous, but *not* isotropic.

The Dynamic Behavior of the C-field

Dynamic systems are often characterized in terms of *impulse response*, suggesting that we need an *impulse* to kick things off dynamically. To achieve such a transient event we invoke ‘*vacuum fluctuations*’. In response to fluctuations, we expect disturbances to occur in the field and propagate, inducing momentum. It is unclear exactly how to characterize the disturbance, so we examine gravitational waves⁵ in flat-space propagating along the z-axis with

$$\bar{h}_{\mu\nu} = \bar{h}_{\mu 0}(t - z), \quad \bar{h}_{\mu 0} = -\bar{h}_{\mu z} \quad h_{\mu\nu} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & h_{xx} & h_{xy} & 0 \\ 0 & h_{yx} & h_{yy} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

where $h_{\mu\nu}$ is considered the wave’s metric perturbation. We focus on the transverse behavior of the field: the plane wave propagates in z , so non-zero components are:⁶

$$\begin{array}{ll} h_{xx} = -h_{yy} & \text{traceless} \\ h_{xy} = +h_{yx} & \text{transverse} \end{array}$$

This last relation $h_{xy} - h_{yx} = 0$ corresponds to angular momentum; *the circulation of the gravitomagnetic field*. Our metric (4) represents a spatially homogeneous dynamic distribution of energy [the field] expanding and contracting anisotropically at different rates in different dimensions. One set of solution parameters $\vec{p} = (-1/3, 2/3, 2/3)$ represents space expanding in two directions, contracting in the third. For simplicity, we will extract the physics of parameters $(p_1, p_2, p_3) = (0, 0, 1)$, representing space expanding in one direction, unchanging in the other two (*relatively* contracting). Such a solution can give rise to non-zero momentum density, serving as the source of curvature represented by n .

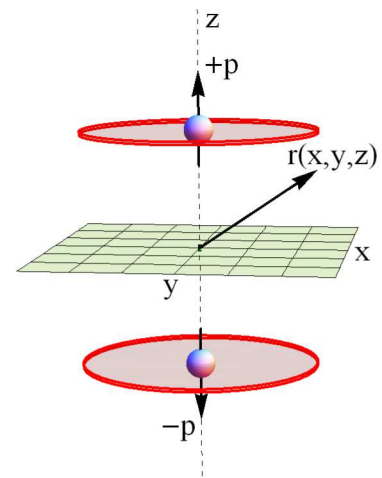


Fig. 1. Physical configuration

Unlike Schwarzschild, Kasner’s solution does *not* represent space *outside* gravitating mass; it represents a dynamic field that expands and contracts in different directions. The Kasner cosmological solution has no *outside*; the expanding field gives rise to momentum \vec{P} . Momentum density $\vec{p} (p_x, p_y, p_z)$ is proportioned according to constraint $Tr(\vec{p}) = 1 = \vec{p} \cdot \vec{p}$. Dimensionless nt implies that parameter n has dimensions of frequency $n \sim t^{-1}$. Vishwakarma establishes the link between n and momentum density \vec{p} via the relation $n \sim \sqrt{gp}/c \sim t^{-1}$. Whereas general relativity makes contact with physical reality by requiring the Newtonian limit $\phi = -M/r$, the matterless link to physical reality is the energy density of the dynamic primordial field. This field momentum density induces the gravito magnetic field \vec{C} whose physical reality

was shown by *Gravity Probe B*¹⁷ and whose theoretical utility was recently shown by Will⁸ to yield Mercury's perihelion advance.

The C-field is identified with angular momentum, $h_{xy} - h_{yx} = 0$; in terms of momentum density \vec{p} the induced gravitomagnetic circulation $\vec{\nabla} \times \vec{C}$ is defined by $\vec{\nabla} \times \vec{C} = -(g/c^2)\vec{p}$ when $\dot{G} = 0$. We assume a dynamical field variation in density such that motion of a denser region through space establishes momentum density \vec{p} , inducing \vec{C} -field circulation. Conservation of momentum invokes an equivalent dense region of field moving in the opposite direction. The vacuum fluctuation inducing momentum flow is initially located at the origin; the direction of momentum density is the z -axis; the xy -plane establishes the symmetry. As there is no *outside*, we predict what will be observed at an *arbitrary point* [excluding the z -axis and xy -plane] located at $\vec{r}(x, y, z)$ with respect to the origin, shown in Fig 1 above. The center of mass of a locally dense region of the field is treated as a point on the z -axis moving with velocity \vec{v} . We relate this change in position to an arbitrary test point \vec{r} and calculate gravitomagnetic field $\vec{C}(\vec{r}, t) \sim \vec{r}' \times \vec{p}$ with $\vec{r} = (x, y, z)$, $\vec{r}' = (x, y, z - vt)$. For equal and opposite momentum density $-\vec{p}$ we calculate \vec{r}'' describing the test point with respect to the second region, moving with velocity $-\vec{v}$. The left-handed C field circulation induced by momentum density in the upper hemisphere circulates opposite to that induced by the downward momentum density:

Evolution of the System

The direction of induced \vec{C} field at arbitrary point $\vec{r} = (x, y, z)$ depends upon the location of the point and the source of the induction. The test point will see *both* induced gravitomagnetic fields therefore we must sum the fields and then square the summed fields $\vec{C} \cdot \vec{C}$ to obtain a value proportional to the gravitomagnetic field energy density at the point.

The geometry of the problem is based on an arbitrary test point \vec{r} from the origin of the disturbance in the field. But since \vec{r} is arbitrary, and the source (momentum density) is moving, what is it we actually wish to calculate? Initially ($t < 0$) there is zero momentum; eventually ($t \gg 0$) the source will have moved far from the test point, therefore we simply display a time-based field energy density observed at \vec{r} . As \vec{r} is excluded from the xy -plane of symmetry, one momentum will first approach \vec{r} , reach a closest point of approach, then recede from \vec{r} , while the other momentum can only ever recede from \vec{r} . The resulting behavior, scaled for convenience, is shown in Fig 2, wherein the horizontal axis is time and the sloped dashed lines represent $z = \pm vt$ (not to scale), while the vertical axis measures the energy density of the C-field at \vec{r} at each point in time. We show the energy contribution from the upward moving field (2a), the downward moving field (2b), and the summed energy, as a function of time (2c).

As a check, we place test point \vec{r} on the xy -plane $\vec{r} = (x, y, 0)$ and run the same calculation. This test point is always equidistant from the source momenta, so the two \vec{C} fields seen at \vec{r} are of equal strength but opposite direction thus they completely cancel at all times: the energy curve (not shown) is proportional to $\vec{C}(\vec{r}, t) \cdot \vec{C}(\vec{r}, t) \equiv 0$. The C-field energy density is given by:⁹

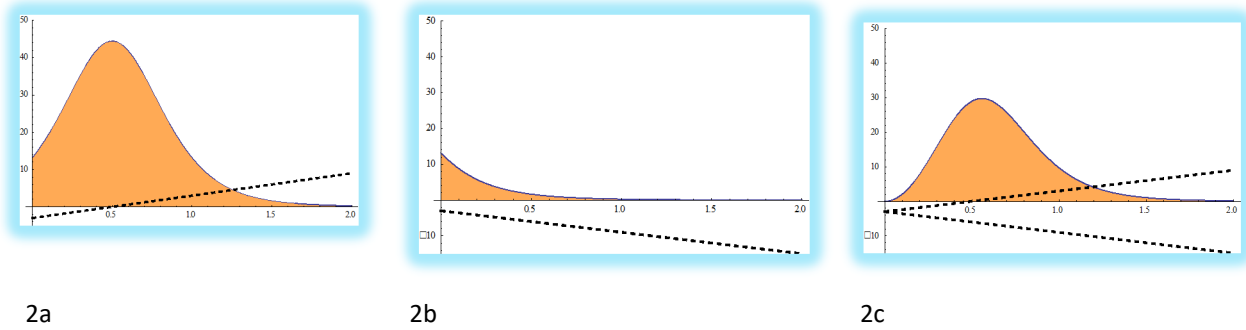


Fig. (2a). Energy density of *left-handed circulation* from positive momentum density at $(0,0,z(t))$ observed at $\vec{r} = (x, y, z)$ beginning at $t = 0$. Momentum flows up the z -axis (dashed lines) reaching a point of closest approach at which field is maximum, then the field declines. (2b) The opposite momentum produces a *right-handed circulation* at \vec{r} , then moves farther away; this contribution always declines. (2c) The two fields are summed and the local field energy density at $\vec{r} = (x, y, z)$ is shown as a function of time.

$$\left(\frac{c^2}{g}\right) \vec{C}(\vec{r}, t) \cdot \vec{C}(\vec{r}, t) = \frac{mv^2}{vol.} = \text{energy density}$$

Our test point is arbitrarily located with respect to the z -axis so any point rotating around the z -axis becomes a *test circle* and we show the time behavior of the field observed at all points on the circle, seen in fig 3:

Energy density at any point in the neighborhood of a disturbance in the field (at the origin) starts at time zero and grows as gravitomagnetic circulation at the point, invoked by the momentum density of the moving region of locally increased energy density, approaching and passing the point at $\vec{r}(x, y, z)$.

At the point of closest approach, the local energy of the induced C-field is maximized; this is a geometrical calculation on a varying geometry. The curves in fig 2 represent the contribution at point $\vec{r}(x, y, z, t) = \vec{r}(\theta, \phi, t)$. By choosing the observation point (x, y, z) we effectively specify a ring of equivalent points, all of which will experience the same induced C-field energy, as depicted in fig 3 along the time axis from $t < \text{event}$ to $t \rightarrow \text{future}$. At $t = 0$ (the event) there is no energy at $\vec{r}(\theta, \phi)$.

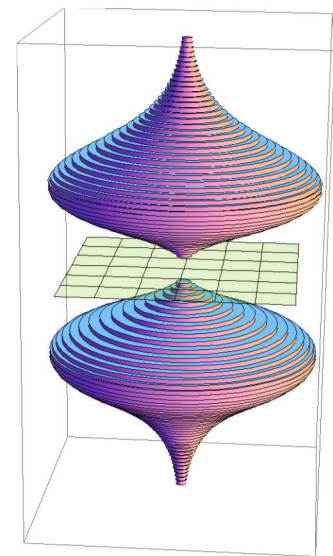


Fig.3. Energy history

As the disturbance in the field propagates along the z -axis, which is also the t -axis, the approaching induction source, momentum density \vec{p} , induces an increasingly strong gravitomagnetic circulation (vorticity)¹⁵ with left-handed circulation increasing and right-handed circulation diminishing at the point. At nearest approach, every point on a circle surrounding $\vec{p}(z)$

and containing $\vec{r}(x, y, z)$ will experience maximum field strength, followed by a decrease as both sources move further away in time and distance. In fig 3 the north-south axis is both *time* t and *distance* z , while the contribution to the C-field induced at point $\vec{r}(x, y, z)$ by momentum $\vec{p}(0, 0, vt)$ peaks at $z = vt$, then diminishes as $vt > z$. Eventually the induced local field energy at $\vec{r}(\theta, \phi)$ effectively vanishes.

We interpret energy-time diagram (Fig 3) by noting that the diagram shows *instantaneous* conditions at \vec{r} over the course of time. There is *no field memory* in our model; such analysis needs still to be performed.

Vishwakarma's n incorporates momentum density as the source of metric curvature; we identify n as the gravitomagnetic field induced by a moving region of higher field density while conserving total momentum (linear and angular). Field density variations break homogeneity: as Kasner is defined as a homogeneous, anisotropic solution, we wish to restore the homogeneous nature to our solution. If initial energy density is unity, the density at the test point becomes $1 + \vec{C}(\vec{r}, t) \cdot \vec{C}(\vec{r}, t)$ over a unit volume, compared to the original density of *one* per unit volume. For a constant density (homogeneous) solution we calculate new volume $C \cdot C_{vol}$ and equate densities:

$$\frac{1 + \vec{C}(\vec{r}, t) \cdot \vec{C}(\vec{r}, t)}{C \cdot C_{vol}} = \frac{1}{vol} \quad \text{therefore} \quad \frac{C \cdot C_{vol}}{vol} = 1 + \vec{C} \cdot \vec{C}$$

and thus *space is expanding at the arbitrary point* $\vec{r}(x, y, z)$:

$$C \cdot C_{vol} = (1 + \vec{C} \cdot \vec{C}) vol$$

This Kasner metric solution of the Einstein field equations, in which one spatial dimension expands while the other two spatial dimensions are static, yields an anisotropic evolution of a homogeneous field.

Conclusion

To a surprising degree, our cosmology at the century mark is unknown; George Ellis¹⁸:

*The nature of the inflaton, the nature of dark matter, the nature of dark energy are **all** unknown.*

Potentially related to our model, Evans & Eckardt¹⁹ claim "*the second Bianchi equation used by Einstein and Hilbert is incomplete*" as is the cosmology based on that equation, except in the narrow special case in which *torsion is zero*. They derive a new identity that, combined with the first Bianchi identity, relates *curvature to torsion*. Their torsion-based vector equations (62) and (63) resemble our C-field equations. And although the source of curvature is typically a stress

energy tensor $T^{\mu\nu}$, *vacuum equations* are seen to have curvature despite a lack of matter. The primordial field is assumed to be the gravitational field, yet for over a century physicists have failed to find a form of $T^{\mu\nu}$ representing the local energy of the gravitational field; Feynman¹⁰ remarked that “*Even for very simple problems, we have no idea how to go about writing down a proper $T^{\mu\nu}$.*” Moreover, the *Horizon problem* concerns the isotropy of the universe based on inflation models wherein separate regions of the universe are never in contact. In this light, our approach based on the Kasner metric, whose physical interpretation has been ‘*obscure*’, suggests a cosmos with rotation. Longo¹² describes his experiment as ‘*testing homogeneity and isotropy*’ via surveys of spiral galaxies, and he finds anisotropy. The 2004 *WMAP axis-of-evil* anisotropy was confirmed in 2013 by the *Planck Telescope* – the *anisotropy is there*.

New Scientist (26 Oct 2016):

From the rotation of galaxies to cosmic expansion everything points in one direction. If only we knew why?

The Kasner metric solution of Einstein's equation describes *how* everything could end up pointing in one direction by virtue of a dynamic primordial event that organizes the field as it evolves. We reject that the source of curvature in Kasner is a singularity appearing at $t = -1/n$, which does *not* appear at any other time, while the solution is curved at *all times*. Our interpretation of *pre-spacetime*, as *primordial field*, implies that *dynamic space-time* corresponds to a dynamic field whose momentum density physically exists. The momentum induces a circulating gravitomagnetic field that distributes field energy locally, effectively *expanding space* at an arbitrary point in space-time. If the field already exists at the given arbitrary point and expansion is inhibited, the local field energy acts to increase expansion *pressure* in the region of the point, potentially acting as *dark energy*. Early in the post-bang era virtually unlimited field energies are expected. The interpretation of Kasner parameter n as the gravitomagnetic field provides a new perspective on the physical nature of the primordial field and a qualitative mechanism and explanation of the evolution of dynamic space-time of the primordial field. The C-field interpretation of the metric induces vorticity subject to conserved momentum; linear and angular. The *time dependence* of the Kasner metric is essentially different from the static *space dependence* of the Schwarzschild and Kerr metrics. This would seem to satisfy a requirement for an evolving space-time theory. Other issues to be investigated for this theory include the nature of the ‘vacuum fluctuation’ that triggers the event, as well as the nature of any field ‘memory’ that sustains circulation after the instantaneous event has ‘moved on’.

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