A Physics-based Disproof of Bell's Theorem

By
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Abstract

After decades of experimental validation, Bell's Theorem has changed the ontological status of local realism in physics. But recent theoretical and experimental results present a new challenge to Bell's analysis. A geometric-algebraic challenge claims that Bell makes a topological mistake, while 'weak measurement' results challenge the Copenhagen Interpretation. We review these results and analyze the physics of Bell's Theorem, embedding Bell's inequality in a truth statement, and showing it to be falsified.

In 1964 at CERN, John Bell[1] developed a new analysis of fundamental quantum mechanics designed to answer questions raised in 1935 by Einstein, Podolsky and Rosen (EPR) [2] concerning the *completeness* of quantum mechanics, defined by:

> *"A complete theory has an element of the theory corresponding to every element of reality."*

At the time, most physicists assumed that the giants of the field (Bohr, de Broglie, Heisenberg, Schrödinger, Born, Dirac, and others) had resolved these issues as well as could be managed. As expressed by David Mermin[3]: quantum mechanics provides a tool for the calculation of probabilities at the atomic and subatomic level of reality, so "Shut up and calculate!" Niels Bohr had convinced most physicists that the human mind was un-evolved and ill-equipped to understand atomic phenomena; that reality manifests itself as *particle-like* or as *wave-like*, depending upon how the question (experiment) is formulated. But experimentalists realized that Bell had actually defined a test that could be performed experimentally and in 1982 Alain Aspect[4], and many other experimentalists since then, have confirmed that:

> *Bell's inequality is consistently violated.*

Yet in 2011 several relevant events have occurred. Joy Christian[5] has presented and defended a theory that claims John Bell chose the wrong topology for his analysis. The complexity of Christian's analysis, combined with the general unfamiliarity most physicists have for geometric algebra, has led to lack of unanimous agreement. Also significant are two new experiments (based on Aharonov's[6] *weak measurement* theory which provides a way around Heisenberg's uncertainty principle) that report results that seem to favor local realism.

Bell's theorem, based directly on the physics discussed by EPR, models the detection of atomic spin using a Stern-Gerlach[7] apparatus. He asks whether the inclusion of 'hidden variables' in the calculations will reproduce the same correlation as quantum mechanics does (without the hidden variables) and finds a different calculated correlation. The problem then becomes one of experimentally measuring these correlations to determine whether "reality" matches the quantum mechanical model or the hidden variable model. The experimental results clearly fall in the quantum mechanics court. The significance of this is that

> *Bell's theorem became* <u>*the*</u> *basis of rejection of local realism in physics.*

Christian addresses *the EPR formulation* of the spinning particles for his topological treatment. He formulates the two-particle problem in both a coordinate-free and a particle-free manner—both *spin-1/2 fermions* and *spin-1 bosons* results agree with reality, hence his *locally real* assumptions appear valid. His analysis applies to both *classical* and *quantum* correlated two-body systems—in every case his calculations agree with reality. Bell computes 2, Christian computes $2\sqrt{2}$, quantum mechanics calculates $2\sqrt{2}$ and experiments produce $2\sqrt{2}$. *Only Bell is wrong!*

Christian claims that a two-particle representation of the Bell Test both *supports local realism* and calculates the correct quantum mechanical results. Yet many are bothered by the fact that Bell's theorem is still mathematically correct. Writing about *hidden variable theories* and *Bell's Test*, Florin Moldoveanu states:

> *"Indeed, Joy did not disprove Bell's theorem as this theorem remains mathematically correct."*

How can Bell be correct *and* Christian be correct? *It's of absolutely no consequence that Bell's theorem is mathematically correct.* The question is whether Bell's theorem is *physically* correct. And it is *not*.

> *So we must analyze Bell's inequality from a physical perspective, not a mathematical perspective.*

We must focus on the logic underlying Bell's inequality. Since even those who accept Christian's mathematical analysis *still agree that John Bell's math is correct*, the issue is not correctness of his math, but correctness of Bell's physics. We reason:

Theory + Experiment = true/false

Theory provides a mathematical *model* or *map* claimed to represent the physical universe in some appropriate manner over a specified region. Experiment attempts to create an appropriate physically real *instance* of the phenomenon in question.

Bell formulated his two-particle problem as spin ½ particles—filtered through Stern-Gerlach apparatus and measured remotely. Before analyzing the problem, let us define our terms. In *On the Einstein-Podolsky-Rosen paradox*, Bell relies upon two settings, $\hat{a}$ and $\hat{b}$ in a Stern-Gerlach experiment designed to measure selected components of the spins $\vec{\sigma}_1$ and $\vec{\sigma}_2$. He then considers the hypothesis that if two measurements are made at places remote from one another, the orientation of one magnet $\hat{a}$ does not influence the result obtained with the other $\hat{b}$. Bell then hypothesizes that a *more complete* specification can be effected by means of parameters $\vec{\lambda}$, with $\rho(\vec{\lambda})$ the probability distribution of $\vec{\lambda}$. The expectation value of the two components $\hat{a}\cdot\vec{\sigma}_1$ and $\hat{b}\cdot\vec{\sigma}_2$ is

$$P(\hat{a},\hat{b}) = \int d\lambda\, \rho(\lambda)\, A(\hat{a},\lambda)\, B(\hat{b},\lambda)$$

which, he claims, should equal the quantum mechanic expectation value, which for the singlet state is

$$< \vec{\sigma}_1 \cdot \hat{a}, \vec{\sigma}_2 \cdot \hat{b} > = -\hat{a}\cdot\hat{b}$$

where A and B are respective results of measurement. Bell associates the parameter λ with '*initial values'* of hypothetical '*hidden variables'* at some suitable instant, including any auxiliary equations.

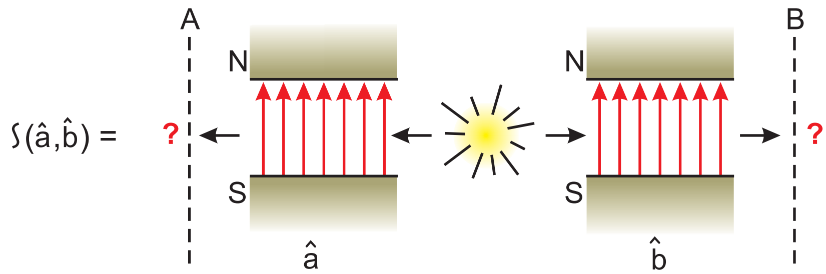
What has Bell proved? He calculated a probability, and used this to generate an inequality

$$f(P(\hat{a},\hat{b})) \quad = \quad |\, P(\hat{a},\hat{b}) - P(\hat{a},\hat{b}')\,| + |\, P(\hat{a},\hat{b}) + P(\hat{a}',\hat{b})\,| \;\leq\; 2$$

In 'real world' experiments, this inequality is violated. This fact has led many to conclude that local realism, the assumption on which Bell based his analysis, does not hold. Yet most discussion of Bell's analysis is purely mathematical in nature. This overlooks the fact that a crucial physical system exists, which we denote by $S(\hat{a},\hat{b})$. Any experiment performed upon this system, combined with Bell's analysis of the experiment, then implies the following:

$$[\; f(P(\hat{a},\hat{b})) \leq 2\;] + S(\hat{a},\hat{b}) \quad \Rightarrow \quad true\,/\,false$$

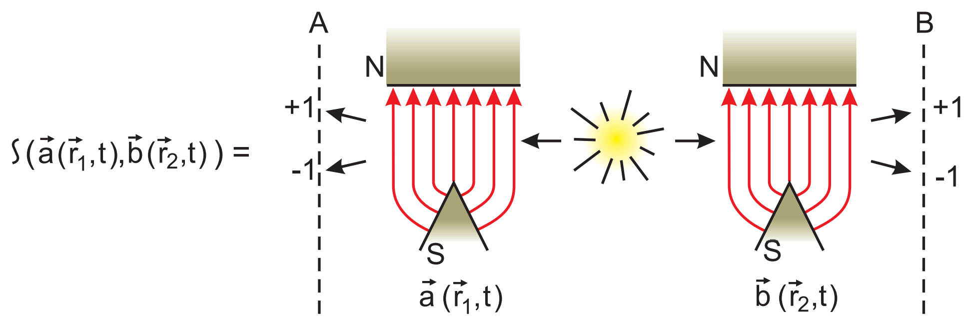
In fact, *this statement is always false*. To understand this we look, not at Bell's math, but at the physics.



In the above diagram we've depicted a physical experiment that corresponds to Bell's analysis. Two magnetic fields are positioned with respective directions $\hat{a}$ and $\hat{b}$ and spin ½ particles traverse these fields, to be detected, respectively, at locations A and B. When this experiment is performed the result is indeterminate—*homogeneous* magnetic fields, corresponding to unit vectors $\hat{a}$ and $\hat{b}$ do *not* split the beam, so no meaningful determination of spin can be made. J.S. Bell of course knew this; in *Speakable and Unspeakable in Quantum Mechanics*[8] he states:

> *"If the field is uniform the net force on the magnet is zero."*

As is well known, in order to split a beam passing through the magnetic field, it is necessary that the field be inhomogeneous, as is the case shown below: [Fields are shown rotated to exhibit non-linearity.]



Therefore the appropriate Bell test should be formulated

$$[\ f(P(\vec{a}(\vec{r}_1,t),\vec{b}(\vec{r}_2,t))) \leq ?\ ] + S(\vec{a}(\vec{r}_1,t),\vec{b}(\vec{r}_2,t)) \quad \Rightarrow \quad true/\,false$$

We can conjecture that the question mark stands for the value $2\sqrt{2}$, since this would match the results of actual experiments performed using a system $S(\vec{a}(\vec{r}_1,t),\vec{b}(\vec{r}_2,t))$, but this is mere conjecture because Bell never calculated this result. Yet even without having calculated the value for the inhomogeneous fields, we have formally disproved Bell's theorem, because the statement

$$[\ f(P(\hat{a},\hat{b})) \leq 2\ ] + S(\hat{a},\hat{b}) \quad \Rightarrow \quad false$$

is indisputable. Bell was not a mathematician aiming to calculate a simple integral and obtain a simple result. Bell was a physicist, bothered by issues that arose with EPR and that had been glossed over for decades. To treat Bell's mathematics and to ignore the physics is a meaningless exercise. The fact is that, given physics corresponding to Bell's mathematics, his statement is false.

> *Results of experiments neither confirm, deny, nor violate his conclusion. All such results are null.*

Some who have invested time and effort in this problem and have hailed the "failure of local realism" as a mystical result may claim that this is a mere quibble—that Bell was merely making an appropriate approximation, as is commonly done in the art of physics. But for almost a half century the most important aspect of physics—

whether or not local realism has meaning—has hung upon Bell's formulation. Moldoveanu[9], in response to Christian, notes:

> *"You say: 'It would be extraordinary if the fate of local realism were to hinge on a technical distinction between vanishing scalars and strictly zero vectors. … [but] the distinction is not technical, but critical.'"*

Similarly, it would be the height of folly to claim that local realism does not exist in our universe based upon a simple approximation that is demonstrably false. To understand this we note that John Bell proceeded by examining a problem that, like most physics problems, included the following:

The relevant dynamical laws
The relevant initial condition (assumed)
The relevant boundary conditions

It is quite common to formulate physics problems and/or design physics experiments such that either the initial conditions or the boundary conditions cancel in some significant way. This often makes the problem simple enough to solve. Bell attempted to ignore the complex boundary conditions of the Stern-Gerlach experimental system by formulating the problem as a homogenous field problem. That may or may not have been an appropriate approximation. But when the experiment was performed, and his result was falsified, there were two possibilities:

1.) go back to the drawing board, or
2.) banish local realism from the universe

For some reason, the physics community made the second choice. The first choice is the correct one. Bell's formulation of the problem is demonstrably incorrect.

The Stern-Gerlach physics is undeniable. Bell's choice of a unit vector, a uniform field, will always produce a null result, literally nullifying the test of his theorem. The inhomogeneous field employed is not described by Bell's mathematical model, and therefore his math is inappropriate for the real experiment. Hence 'violation of Bell's inequality' is meaningless, since theory and experiment are incompatible.

But what about photons? Although Bell formulated his theory for Stern-Gerlach (corresponding to the EPR formulation that he was addressing) many relevant experiments are actually performed not on spin ½ particles but on polarized photons. How does this affect the above arguments? The physics involved might seem to more closely approximate the unit vector descriptors, $\hat{a}$ and $\hat{b}$. Yet even this case is not so simple. Factors involved include:

- 'Changes en route' can violate inequality
- 'Non-linear effects' occur in ALL photon experiments
- 'Entanglement Sudden Death' (ESD) in fiber optics
- 'Detector inefficiency' and 'randomness' issues
- Christian's analysis *included* photons – same $2\sqrt{2}$ result.

But the fact is that en route changes occurring before the measurement is made—in either branch—will affect the results and may lead to violation of the inequality without in any way challenging the concept of local realism. Such *changes en route* are implicitly boundary condition effects having nothing to do with initial conditions. So the need to include boundary conditions in calculating the inequality exists.

We can examine the issue of *initial conditions* versus *boundary conditions.* Whereas the physical re-arrangement of magnets occurs mechanically, the polarizing filters can be operated at electronic speeds. And one of the 'holes' in the experiment, guaranteed randomness, is addressed by 'delayed choice' experiments in which the filter angle is chosen (as randomly as possible) only at the last moment, in order to require super-luminal communications, if any occur. The *last minute* imposition of a random polarizing filter orientation is absolutely *not* to be considered an initial condition. It is a time-dependent boundary condition. This should be obviously true from the mere description of the experiment. But it's also implied by the actual results of the experiment:

> *As long as the same polarization angles are applied to each path, the correlation is perfect. Only when different angles are chosen for each path does the correlation change. From this we infer that identical boundary conditions cancel, but varying boundary conditions must be handled appropriately!*

It is remarkable that, as long as both particles are subjected to exactly the same test, the correlation is perfect. Here initial conditions are sufficient, since boundary conditions cancel. Only when different filter operations are performed on each path does the correlation behave in a more complex manner. As a result of further analysis, most recent experiments use 'delayed choice' as follows. In order to eliminate the possibility that the state of A's filter is somehow known by B, the state of A's filter is determined, randomly, only at the last moment before the measurement is made (and the same for B's filter). The changes are ideally made in such manner that knowledge of the other filter would require super-luminal transmission of information. As a consequence, there is absolutely no way that the 'delayed choice' state of the filter operation can be considered an *initial condition*. It is a *boundary condition* that is a function of time. So we must replace $\hat{a}$ and $\hat{b}$ by $\vec{a}(\vec{r}_1,t)$ and $\vec{b}(\vec{r}_2,t)$ subject to $\delta[\vec{a}(\vec{r}_1,t)]\neq 0$ and $\delta[\vec{b}(\vec{r}_2,t)]\neq 0$.

But does the centrality of inhomogeneity to the spin ½ cases apply to photons? If not, the use of $\hat{a}$ and $\hat{b}$ as polarizations may be appropriate approximations. The *changes en route* argument is independent of this representation. And all real photon experiments *require* non-linearities in the photon paths in order to affect the experiment. Finally, Christian has shown[18] that a correct coordinate-free geometric model of the physical space calculates the quantum mechanical result.

The above exposition shows that *attempts to banish local realism from our physical universe are premature at best*. Local realism is compatible with the fascinating results obtained in the last year or so. Results compatible with a <u>*particle AND wave*</u> theory are quite mysterious from a <u>*particle OR wave*</u> (Copenhagen) perspective.

What's at stake?

From a human perspective, the stakes are the highest possible. The Bohr, or 'mystical', tradition celebrates the 'weirdness' of quantum mechanics and says, in essence, that our brains are not equipped to deal with reality. The classical tradition of local realism assumes the human ability to 'intuit' reality.

Bell's inequality has tilted the scales heavily against local realism, If Bell is wrong, based on Christian's topological analysis—or based on the simple inappropriateness of his formulation of the problem, as I contend—or for any other reason, then local realism has gotten a bad rap.

Despite much discourse, Christian's approach is not yet generally accepted. It is indisputable that uniform fields corresponding to Bell's formulation cannot possibly produce Bell's results—though I expect it to be disputed. Arguments against Bell are somewhat subtle; experiments, from Aspect to Zeilinger[10], seem more conclusive. So it is paramount to understand that <u>all</u> experiments that violate Bell's inequality are *meaningless* if Bell made a logical mistake.

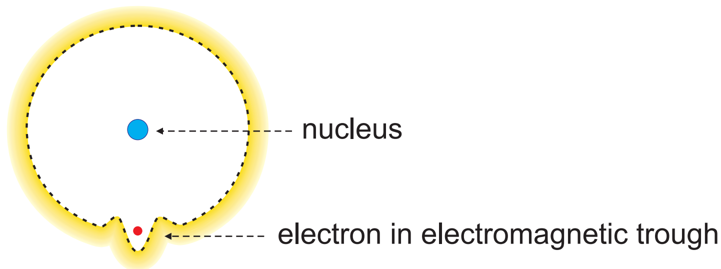
Even better would be experiments that support local realism. And within the last few months just such experiments have been reported. We look at these next.

Experimental Proof

The Michelson-Morley experiment showed no effects of *the ether*, the Eddington experiment measured the *bending of light* predicted by general relativity, and the Davisson-Germer experiment showed the *wavelength of particles*. All of these, we are told, *settled the issue*. Somehow, today, experiments do not so readily settle issues. Experiments have been ongoing for 3 decades that *violate Bell's inequality* and still there are questions about "holes" in the experiments[11] (most having to do with *detector efficiency*, but also with *randomness*). It doesn't help that they rely upon statistical correlations, as opposed to simply exhibiting physical phenomena.

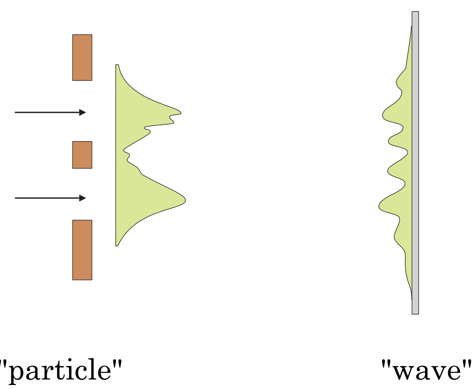
Similarly, *hints of Higgs* have been reported at the Tevatron and LHC. The question is whether 'real particles' are being seen or simply statistical artifacts that are attributable to the analysis software. Yet experiments that seem rather clear cut haven't been generally accepted. For example, Martin Tajmar's experimental detection[12] of the C-field has made little discernable impression on the physics community. Likewise, the 4% anomaly[13] in the *proton radius* determined by muonic-hydrogen measurements. Even 120 orders of magnitude error in vacuum energy seems to have had little impact on QED and QCD. So it probably shouldn't be surprising that recent evidence for the reality of *particle AND wave* (vs. the Copenhagen belief system's *particle OR wave*) has as yet had little impact.

One such experiment shows *non-dispersing electrons in a Bohr orbit*. Since the days of Heisenberg's uncertainty principle it's been considered to be the case that orbital electrons *disperse,* existing only as a *probability density*. But corpuscular electrons can attribute such non-localized behavior to random noise, from cosmic microwave background radiation or local radio and TV transmissions. This however is not the view of Copenhagen physicists, who view particles as *superpositions of wave functions* rather than as locally real particles. Yet Maeda[14] et al. have created a "traveling trough" and found that an orbital electron will travel in this trough without dispersing, *just like a 'real' particle.*



Red electron travels in yellow electromagnetic trough orbiting blue nucleus.

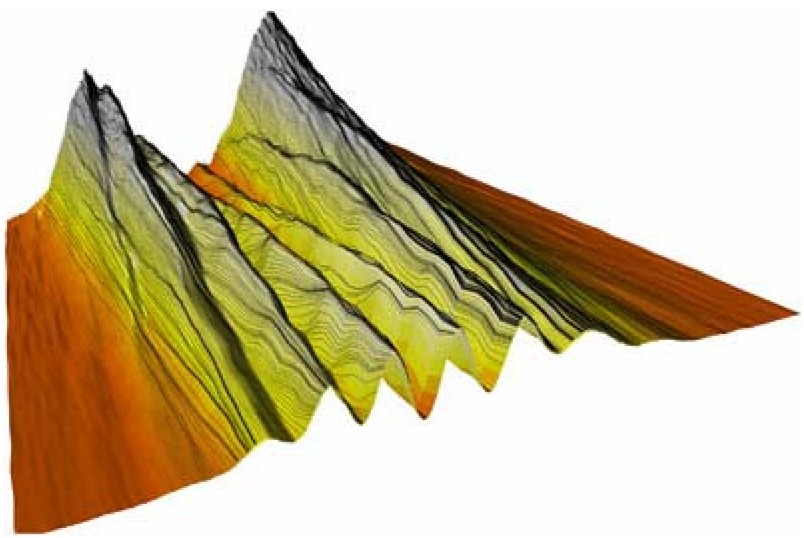
For most of the history of quantum mechanics the "two slit experiment" has been a touchstone. When particles are measured at the slits, they appear to be particles that came through one slit or the other. But measurements far from the slit appear to show wavelike interference. The Copenhagen *'particle or wave*' interpretation can make little sense of this, and does *not* support the idea of 'particle trajectories'.



The de Broglie-Bohm *'particle and wave*' interpretation *does* entail particle trajectories. Due to the popularity of the Copenhagen view and the more recent violations of Bell's inequality, the idea of "real particles" has been losing out for decades. And until very recently the idea of experimental tests seemed severely constrained by Heisenberg's uncertainty principle. Much obfuscation has grown up around this fact and has been attributed to *Heisenberg's uncertainty* but the simple fact is that the measurement of momentum implies a change in position (Δx) and this cannot be obtained in one measurement. There are many mathematical *explanations* for this, but *change* always requires two measurements, and prior experimental concepts and techniques were such that the first measurement disturbed the system in such a way that the following measurement was compromised.

Quantum measurements historically produced strong results that appeared on a classical apparatus. The cost of this appearance was the disturbance of a coherent quantum system. This experimental picture changed in 1988 due to Aharonov's idea of *weak measurements* in which a first measurement is too weak to disturb the state of the system, but is followed by a stronger measurement that does perturb the system. Such experiments are now producing data that was previously believed impossible to come by. Aharonov's *weak measurement* does not produce an unambiguous classical state of the apparatus, but is followed by a *strong measurement* that records the quantum state. By measuring sequentially two complementary variables of the system, one can obtain information that is forbidden in principle to one measurement. The first measurement provides correspondingly less precision, but this precision can be regained by averaging.

The *weak measurement* provides information on one property "*without disturbing the complementary property and so the future evolution of the entire system*". An ensemble of such measurements allows statistical reconstruction of behavior that was deemed impossible to obtain. In one of the most recent examples Aephraim Steinberg[15] et al. have managed to measure momentum and position of single photons in a two slit experiment. A weak or imprecise measurement of momentum was followed by an accurate measurement of final position (where the photon hit the camera). Their incredible 3-D plot of a single photon is shown below:



What is of particular interest is Steinberg's statement that

> *"Our measured trajectories are consistent… with the realistic but unconventional interpretation of quantum mechanics[16] of such influential thinkers as David Bohm and Louis de Broglie".*

Make no mistake; this is a landmark quantum mechanics experiment. For a century the *principle of complementarity* devised by Bohr has used the two slit experiment to teach that either particle properties or wave properties can be observed, but the system cannot behave as both a particle and wave simultaneously. Steinberg has proved that this doesn't have to be the case—the system can behave as both.

The particle is simultaneously wave and corpuscle!

Since 1926 the Schrödinger wave function has been the central mathematical entity in quantum mechanics, believed to completely specify the state of the system. Yet in the Copenhagen Interpretation, *the complex-valued wave function is not itself considered to be a physical element of quantum theory*. For this reason it has been believed that the unknown wave function of a system cannot be determined experimentally, even in principle, for a quantum system.

As a way around this, *tomographic* techniques perform measurements upon an ensemble of quantum systems to determine the statistics, then, having obtained a measure of probability, attempt to extract a probability amplitude algorithmically, an 'indirect' measurement of the wave function.

Yet Jeff Lundeen[17] developed the concept and theory for direct measurements of the quantum wave function and in 2011 reported results of experiments conducted by his team. The effectively *orthogonal* information derived by Aharonov's technique can be represented as *real* plus *imaginary* information and represented as the quantum wave function. This *direct measurement* lends new reality to the wave function and demands an explanation hitherto satisfied by 'mystical' claims.

It is difficult to overstate the significance of this. Lundeen has succeeded in directly measuring the physical reality of the quantum mechanical wave function that most physicists have believed to be essentially non-physical in some mystical sense. As John Duffield comments in *PhysicsWorld*:

> *"It's good to see the quantum-mysticism 'you can never understand it' edict being shot down by real experimentation which gingerly feels out the photon wave function."*

Incredibly, Lundeen's direct measurement of the wave function was reported in *Nature* (9 Jun 2011), the same week that Steinberg's report of photon trajectories through a two-slit apparatus was reported in *Science* (3 Jun 2011). Also remarkable is the complementary nature of the experiments, to wit:

Steinberg: Weak momentum → strong position measurement.

Lundeen: Weak position → strong momentum measurement.

Also remarkable is that Lundeen's weak quantum measurements of the transverse spatial wave function "*directly probes the real and imaginary parts of the wave function of the ensemble*". The treatment of the *imaginary number* ($i = \sqrt{-1}$) is here simply a shorthand for *orthogonal*. In static systems three Cartesian coordinate axes easily express orthogonality. But in a rotating system the relative components are constantly changing. What is constant is the orthogonality. Thus $i = \sqrt{-1}$ is simply a means of combining two equations into one equation.

Conclusion

For almost a century quantum physics has trended toward more mystical, beginning with Niels Bohr's *principle of complementarity* and the Copenhagen *superposition of wavefunctions* and *collapse of the wave function* and gathering steam with Aspect's and others experimental *violation of Bell's inequality*. What were originally assumed to be locally real phenomena have evolved to *non-local, non-real properties* that only collapse upon measurement and now the results of measurement reflect remote (non-local) information conveyed super-luminally. Joy Christian’s geometric-algebraic treatment of Bell's Test calls this interpretation into question.

I’ve chosen to ignore Christian’s math problem to re-direct focus to the physics problem, which is that Bell attempted to use incorrect boundary conditions and thereby derived what is in essence nonsense, This led to non-local, non-real concepts that increased the 'mysticism' at the expense of clarity. But physicists can not ignore Steinberg's demonstration of particle trajectories and Lundeen's direct measurement of the quantum wave function.

I have shown why Bell was wrong. Joy Christian has shown how *coordinate-free parallelizable* representations yield the same results as quantum mechanics, and Aspect-to-Zeilinger have experimentally shown that the QM calculation is correct. Finally, Steinberg and Lundeen have, separately and convincingly, shown that the deBroglie interpretation of *particle AND wave* is the reality, and the almost century-old Copenhagen interpretation is mistaken.

Despite Christian's calculation of topologically correct correlation, there is no theory explaining how probability amplitudes (wave functions) relate to *particle AND wave* phenomena. I have worked out this theory and hope to present it this year. (2011)

Edwin Eugene Klingman

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